An Electromagnetic Field and Electric Circuit Coupling Method for Solid Conductors in 3-D Finite Element Method

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***Abstract*—Traditional low-frequency eddy-current solvers do not include stray capacitive effects while high-frequency solvers do not take into account of the internal regions of solid conductors and external circuit excitations. In this paper the application of the edge-element method to problems with both inductive and capacitive effects for solid conductors under external circuit excitations using the formulation with a magnetic vector potential and an electric scalar potential is presented. A novel field-circuit coupling method is presented, which has the advantage of being convenient in algorithm implementation. A numerical example is given to showcase the proposed formulation and the developed edge-element program.**

***Index Terms*—Displacement current, edge-element, field-circuit coupling, finite element method, solid conductor, three-dimensional field.**

I. INTRODUCTION

To address distributed parasitic effects in electromagnetic (EM) devices, such as the coupled inductive and capacitive effects in complicated solid conductors in high-frequency transformers or electric machines driven by power electronics, the displacement current has to be considered [1-3]. As traditional low-frequency finite element (FE) solvers do not include displacement current while high-frequency solvers do not address nonlinearities in the materials and solutions inside solid conductors, it is very useful if a new solver can be developed to tackle these problems. As EM devices are usually driven by external circuits, the development of a field-circuit coupling method is thus of top priority.

Edge-elements, also known as vector elements, are widely used nowadays to approximate vector fields in electromagnetics because of their proper physical sense. Edge-element is a natural choice for discretizing vector fields as it is relatively accurate in solving field discontinuities between the interfaces of air and iron materials [4].

In this paper, the edge-elements and their applications for problems with inductive, capacitive and external circuit excitations using the magnetic vector potential (MVP) formulation is presented. A novel EM field and electric circuit coupling method in regions of solid conductors is presented. The proposed algorithm has the merits in that it is not necessary to search along the integration path of the EM force. The fully-discretized FE scheme is given in details. Numerical result is given to verify the proposed formulation and the developed edge-element program. The stability and feasibility of the developed solver is highly useful when it is necessary to simultaneously consider coupled inductive and capacitive effects with external circuits in EM devices.

II. PROPOSED FORMULATION

A. MVP Formulation for Full Wave Maxwell Equations

The full-wave Maxwell system reads

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot \vec{B} = 0, \tag{3}$$

with constitutive equations or material laws

$$\vec{B} = \mu \vec{H}, \tag{4}$$

$$\vec{J} = \sigma \vec{E}. \tag{5}$$

From (3), one can introduce the MVP $\vec{A}$ satisfying $\nabla \times \vec{A} = \vec{B}$; from (2), one can introduce the electric scalar potential (ESP) φ such that

$$\vec{E} = -\partial \vec{A} / \partial t - \nabla \varphi \ . \tag{6}$$

Then from (1) and the continuity equation of the total current, the MVP ($\vec{A}$ - φ) formulation for the full wave Maxwell problems is stated as

$$\begin{cases} \nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \dfrac{\partial \vec{A}}{\partial t} + \sigma \nabla \varphi + \varepsilon \dfrac{\partial}{\partial t}\left(\dfrac{\partial \vec{A}}{\partial t} + \nabla \varphi \right) = \vec{J}_s \\ \nabla \cdot \left(-\sigma (\dfrac{\partial \vec{A}}{\partial t} + \nabla \varphi) - \varepsilon \dfrac{\partial}{\partial t}\left(\dfrac{\partial \vec{A}}{\partial t} + \nabla \varphi \right) \right) = 0 \end{cases}, \tag{7}$$

For the discretization of the vector field $\vec{A}$, an edge-element is used; while for the discretization of the scalar field φ, a nodal element is applied. In practice, if one directly discretizes the second-order temporal derivative in (7), such as by using the Newmark scheme [5], the resultant scheme may become unstable. As proposed in [6], one can neglect this term when the effects of wave propagation and radiation are negligible. The control equations for problems with both inductive and capacitive effects in time-domain then become:

$$\begin{cases} \nabla \times \left(\nu \nabla \times \vec{A} \right) + \sigma \dfrac{\partial \vec{A}}{\partial t} + \sigma \nabla \varphi + \varepsilon \dfrac{\partial \left(\nabla \varphi \right)}{\partial t} = \vec{J}_s \\ \nabla \cdot \left(-\sigma (\dfrac{\partial \vec{A}}{\partial t} + \nabla \varphi) - \varepsilon \dfrac{\partial \left(\nabla \varphi \right)}{\partial t} \right) = 0 \end{cases}. \tag{8}$$

B. Method to Couple Field with External Circuit

In practice, EM devices are usually driven by external circuits as illustrated in Fig. 1. In this paper a field-circuit coupling method for solid conductors is proposed.

For solid conductors with external circuit excitation, one can introduce two degree-of-freedoms (DoFs), namely the terminal voltage difference *Vs* and the total current *I* for each conductor, and the fully-discretized finite element scheme can be expressed as

$$\begin{bmatrix} \nu K+\frac{\sigma M}{\Delta t} & \sigma K_{AV}+\frac{\varepsilon K_{AV}}{\Delta t} & 0 & 0 \\ \frac{\sigma}{\Delta t}K_{VA} & \sigma K_{VV}+\frac{\varepsilon K_{VV}}{\Delta t} & 0 & 0 \\ \cdots & \cdots & 1 & 0 \\ 0 & 0 & R_e+\frac{L_e}{\Delta t} & 1 \end{bmatrix} \begin{bmatrix} A^n \\ \varphi^n \\ I^n \\ V_s^n \end{bmatrix} = \begin{bmatrix} \frac{\sigma M}{\Delta t} & \frac{\varepsilon K_{AV}}{\Delta t} & 0 & 0 \\ \frac{\sigma K_{VA}}{\Delta t} & \frac{\varepsilon K_{VV}}{\Delta t} & 0 & 0 \\ \cdots & 0 & 0 & 0 \\ 0 & 0 & \frac{L_e}{\Delta t} & 0 \end{bmatrix} \begin{bmatrix} A^{n-1} \\ \varphi^{n-1} \\ I^{n-1} \\ V_s^{n-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ U \end{bmatrix}, \tag{9}$$

where the last but one equation is discretized from

$$I=-\iint_S \sigma\left(\frac{\partial A}{\partial t}+\nabla\varphi\right)dS, \tag{10}$$

and the last equation is discretized from the voltage balance equation, where U is the applied voltage:

$$U=Vs+R_eI+L_e\frac{dI}{dt}. \tag{11}$$

III. Numerical Examples

A. Example 1

For this example, a capacitor with an alternating voltage excitation is numerically solved in the time-domain, as shown in Fig. 2. The radius of the upper terminal of the iron conductor is 1mm; the radius and height of the cylindrical dielectric are 5mm and 0.5mm, respectively. The conductivity of the iron conductor is 10^6S/m.

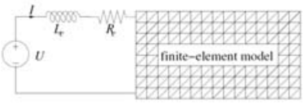

Fig. 1. Illustration of the field-circuit coupling problems.

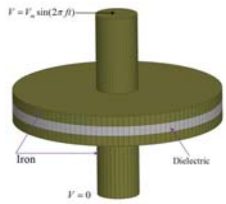

Fig. 2. The capacitor with alternating voltage excitation.

With the excitation peak voltage V_m=1000V, f=10^4Hz, the relative permittivity of the dielectric material being 10^4, the total terminal current versus time-steps is given in Fig. 3.

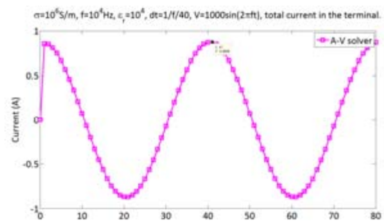

Fig. 3. The current flowing out of the terminal of the capacitor.

For the capacitor being considered, one can easily calculate the expected current as

$$\begin{aligned} I &= C\frac{dV}{dt}=\frac{\varepsilon_r\varepsilon_0 S}{d}\frac{dV}{dt} \\ &= \frac{10^4\varepsilon_0\pi(0.005)^2}{0.0005}\cdot 1000\cdot 2\pi\cdot 10^4\cos(2\pi\cdot 10^4 t), \\ &\approx 0.87\cos(2\pi\cdot 10^4 t) \end{aligned}$$

which can be observed from Fig. 4, where the peak current is 0.8699A and the current is of cosine wave form, which therefore validates the proposed formulation (8).

B. Example 2

For this example, a simple transformer similar to that in [7] is studied, where the primary solid winding is connected with an external voltage excitation and the secondary winding is connected to a parallel capacitor. The computed magnetic flux density is shown in Fig. 5 at the time of 2.5×10^{-3}s. In the full paper, more results will be given to showcase the effectiveness of the proposed method.

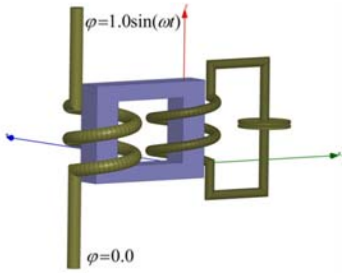

Fig. 4. The geometry of the studied transformer with AC voltage excitation.

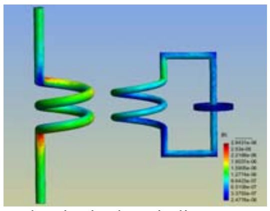

Fig. 5. The magnetic flux density in the windings.

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